

# Constrained twistors principle of the string theory

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## Abstract

The new principle of constrained twistor-like variables is proposed for construction of the Cartan 1-forms on the worldsheet of the  $D = 3, 4, 6$  bosonic strings. The corresponding equations of motion are derived. Among them there are two well-known Liouville equations for real and complex worldsheet functions  $W(s, \bar{s})$ . The third one in which  $W(s, \bar{s})$  is replaced by the quaternionic worldsheet function is unknown and can be thought of as that of the  $SU(2)$  nonlinear  $\sigma$ -model governing the classical dynamics of the bosonic string in  $D = 6$ .

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# 1 Introduction

Let us remind that in the geometrical approach to string theories the classical dynamics of  $D = 3$  and  $D = 4$  relativistic strings is described by the real and complex Liouville equations for the corresponding worldsheet function  $W(s, \bar{s})$  [1]. These equations are achieved by solving for the Virasoro gauge constraints in terms of independent physical string variables restricted only by the embedding conditions. The latter are imposed to insure that the string worldsheet would be a minimal surface in a target space-time. It is worth mentioning that the embedding procedure can be regarded as a further restriction of the string coordinates which involves the second order worldsheet derivatives. It has been observed by Barbashov, Nesterenko and Chervyakov ( BNC in what follows ) [2] that only when these constraints are taken into account together with the Virasoro ones, the number of the string coordinates in a  $D$ -dimensional space-time are reduced to the  $2(D - 2)$  independent variables. Just in that case the system of the Maurer-Cartan equations which provides the embedding of the worldsheet surface into the target space-time is reduced to the Liouville equation in the dimensions 3 and 4.

In this letter we would like to note that there are nice solutions for both of the Virasoro and BNC constraints in terms of the twistor-like variables subjected to the first order derivatives conditions. It will be shown that these twistor constraints are a new basic ingredients of any geometrical approach to a string theory, which at least in the cases of lower non-critical dimensions allows one to gain a deeper insight into the structure of strings.

The paper is organized as follows.

In Section 2 for the convenience of the reader we deal with the first-hand view of the alternative formulation of already known examples for the  $D = 3$  and 4 strings. We show how with the help of constrained twistors principle one can get all the relevant geometrical objects of the corresponding worldsheet omitting the tedious technical details of calculations inherent to the traditional approaches [3].

The generalization of these results to the case of six space-time dimensions is developed in Section 3. Here the new equation of motion for the  $D = 6$  strings is derived in terms of the quaternionic worldsheet function  $W(s, \bar{s})$ . The latter turns out to be composed of the constrained twistor-like variables which are represented by the spinors of the group  $SU(4) = SL(2, H) \sim SO(1, 5)$ . It is amusing that the l.h.s. of the new equation strongly resembles the ordinary equation of the nonlinear  $\sigma$ -model for the principal field  $G = \exp(-W_0 - i\vec{W}\vec{\sigma})$  taking its values on the  $SU(2)$  group.

## 2 Constrained twistor-like variables in $D = 3, 4$

It is well-known that in  $D = 3$  space-time the string coordinates  $x^m(s, \bar{s})$ ,  $m = 0, 1, 2$  are restricted by the following equation of motion and covariant constraints [1, 2]:

$$\partial_s \partial_{\bar{s}} x^m = 0 \quad (1)$$

– the equation of motion,

$$\partial_s x^m \partial_s x_m = \partial_{\bar{s}} x^m \partial_{\bar{s}} x_m = 0 \quad (2)$$

– the Virasoro conditions,

$$\begin{aligned} \partial_s^2 x^m \partial_s^2 x_m &= -4q^2, \\ \partial_{\bar{s}}^2 x^m \partial_{\bar{s}}^2 x_m &= -4\bar{q}^2 \end{aligned} \quad (3)$$

– the BNC constraints, where  $q$  and  $\bar{q}$  are the real parameters of dimension  $+1$ . It is easy to see from (1-3) that there exists the set of commuting  $SL(2, R)$  spinors  $v^\mu(s)$ ,  $u^\mu(\bar{s})$ ,  $\mu = 1, 2$ , which allows one to resolve these constraints in form

$$\begin{aligned} \partial_s x^{\mu\nu} &= v^\mu v^\nu, & \partial_{\bar{s}} x^{\mu\nu} &= u^\mu u^\nu, \\ v_\mu \partial_s v_\nu - v_\nu \partial_s v_\mu &= q \varepsilon_{\mu\nu}, \\ u_\mu \partial_{\bar{s}} u_\nu - u_\nu \partial_{\bar{s}} u_\mu &= \bar{q} \varepsilon_{\mu\nu}, & \varepsilon^{12} &= 1. \end{aligned} \quad (4)$$

The conventions in (4) are as follows

$$x^{\mu\nu} = \frac{1}{2} x^m \gamma_m^{\mu\nu} \quad (5)$$

with the symmetric  $SL(2, R)$  matrices

$$\gamma_m^{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (6)$$

Now it is not difficult to verify that the worldsheet function  $W(s, \bar{s})$  which is defined with the relation

$$u_\mu(\bar{s}) v_\nu(s) - u_\nu(\bar{s}) v_\mu(s) = \varepsilon_{\mu\nu} \exp[-W(s, \bar{s})] \quad (7)$$

satisfies the Liouville equation

$$\partial_s \partial_{\bar{s}} W = q \bar{q} \exp(2W). \quad (8)$$

Indeed, let us introduce the set of the harmonic variables  $v_\mu^\pm(s, \bar{s})$  instead of chiral ones:

$$\begin{aligned} v_\mu^+(s, \bar{s}) &= \exp\left(\frac{1}{2}W\right) u_\mu(\bar{s}), \\ v_\mu^-(s, \bar{s}) &= \exp\left(\frac{1}{2}W\right) v_\mu(s), \\ v_\mu^+ v_\nu^- - v_\nu^+ v_\mu^- &= \varepsilon_{\mu\nu}. \end{aligned} \quad (9)$$

Then directly from the constraints (4) we get the following quantities

$$\begin{aligned}\Omega^{--} &\equiv v^{-\mu} dv_{\mu}^{-} = q ds \exp(W), \\ \Omega^{++} &\equiv v^{+\mu} dv_{\mu}^{+} = \bar{q} d\bar{s} \exp(W), \\ \Omega^0 &\equiv v^{+\mu} dv_{\mu}^{-} = -\frac{1}{2}(ds \partial_s - d\bar{s} \partial_{\bar{s}})W,\end{aligned}\tag{10}$$

which are easily recognized as the differential one-forms that appeared in the framework of a standard geometrical approach to a string theory [3, 4, 5]. Thus, all the results of this approach can be recast in the harmonic representation (4),(9). In particular, to recover the Liouville equation (8) it is sufficient to notice that the forms  $\Omega^{\pm\pm, 0}$  as it stands in (10) satisfy the  $SL(2, R)$  Maurer-Cartan equations

$$d\Omega^{\pm\pm} \pm 2\Omega^0\Omega^{\pm\pm} = 0,\tag{11}$$

$$d\Omega^0 + \Omega^{++}\Omega^{--} = 0.\tag{12}$$

The first two equations (11) are satisfied identically when the r.h.s. of equations (10) are taken into account while the third one (12) is equivalent to (8).

Thus we have constructed a new version of the geometrical approach to a string theory in  $D = 3$ . The guiding principle of this formulation was the twistor equation (4) where the twistor-like variables  $v^{\mu}(s)$ ,  $u^{\mu}(\bar{s})$  and its first derivatives are combined as the components of the relativistic basis. We have right to anticipate that this formulation will be useful in the cases of higher dimensions too.

To be more precisely let us consider the string in  $D = 4$  in brief outline. In this case as it is well-known the Lorentz group is isomorphic to the  $SL(2, C)$ . Accordingly, the constraints (4) can be rewritten in terms of the  $SL(2, C)$  Van der Warden spinors  $v^{\mu}(s)$ ,  $u^{\mu}(\bar{s})$  and their conjugated  $\bar{v}^{\dot{\mu}}(s)$ ,  $\bar{u}^{\dot{\mu}}(\bar{s})$ :

$$\begin{aligned}\partial_s x^{\mu\dot{\mu}} &= v^{\mu}(s)\bar{v}^{\dot{\mu}}(s), & \partial_{\bar{s}} x^{\mu\dot{\mu}} &= u^{\mu}(\bar{s})\bar{u}^{\dot{\mu}}(\bar{s}), \\ v_{\mu}\partial_s v_{\nu} - v_{\nu}\partial_s v_{\mu} &= q \varepsilon_{\mu\nu}, & u_{\mu}\partial_{\bar{s}} u_{\nu} - u_{\nu}\partial_{\bar{s}} u_{\mu} &= \bar{q} \varepsilon_{\mu\nu}, \\ x^{\mu\dot{\mu}} &= \frac{1}{2}x^m \sigma_m^{\mu\dot{\mu}}, & m &= 0, 1, 2, 3.\end{aligned}\tag{13}$$

The only difference of equation (13) in comparison with (4), (5) and (6) is that the parameters  $q, \bar{q}$  become complex and the matrices (6) are replaced by the relativistic  $SL(2, C)$  matrices  $\sigma_m^{\mu\dot{\mu}}$ .

Their relation to all the other equations (7) - (12) of our formulation is just the same as in the case of  $D = 3$  space-time. Moreover, the Liouville equation (8) is still unchanged too but

with the complex worldsheet function  $W(s, \bar{s})$  defined by the expression formally coinciding with (7).

Notice, that as in the case of  $D = 3$  considered previously, the chiral twistor-like variables  $v_\mu(s)$ ,  $u_\mu(\bar{s})$  are restricted by the equation (13) involving its first derivatives. Once again this equation turns out to be decisive in reducing the dynamics of string to the nonlinear Liouville equation.

In the next Section similar equation will be used to extend this approach to more complicated case of  $D = 6$  string.<sup>1</sup>

### 3 Bosonic string in $D = 6$ as a nonlinear $\sigma$ -model

We begin this section with some preliminary remarks concerning our notations and conventions.

In the case we are dealing with the string coordinates which are represented by the anti-symmetric matrix

$$x^{\mu\nu} = -x^{\nu\mu} = x^m \gamma_m^{\mu\nu} \quad (14)$$

with the  $\gamma$ -matrices satisfying the conventions [6, 7]

$$\begin{aligned} \gamma_{\mu\nu}^m \gamma^{n\mu\nu} &= -4\eta^{mn}, & \eta &= \text{diag}(+ - - - -), \\ \gamma_{\mu\nu}^m \gamma_{m\rho\sigma} &= -2\varepsilon_{\mu\nu\rho\sigma}. \end{aligned} \quad (15)$$

Respectively, the underlying twistor-like variables are introduced as the four-component Weyl spinors  $v_i^\mu(s)$ ,  $u_i^\mu(\bar{s})$  with  $SU(4) = SL(2, H) \sim SO(1, 5)$  Lorentz index  $\mu = 1, 2, 3, 4$  and an extra  $SU(2)$  doublet index  $i = 1, 2$ . Note that because of the pseudoreality condition

$$\overline{v_i^\mu} = C^\mu_\nu \varepsilon^{ij} v_j^\nu, \quad (16)$$

where  $C^\mu_\nu$  is the charge conjugation matrix, the spinor  $v_i^\mu$  has primarily eight real components. But only half of them may be considered as truly independent.<sup>2</sup> The rest ones can be eliminated due to the orthogonality condition

$$T_i^j \equiv v_i^\mu v_\mu^j = 0. \quad (17)$$

To prove the last statement let us consider the  $SU(2)$  invariant vector

$$v^{\mu\nu} = \varepsilon^{ij} v_i^\mu v_j^\nu. \quad (18)$$

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<sup>1</sup> The case of  $D = 5$  string is excluded because it yields the known system of equations which has been discussed earlier (see e.g. [1]).

<sup>2</sup> See also discussion in [7].

This vector is light-like because from the rules of the raising and lowering the pair of antisymmetric  $SU(4)$  indices [6]

$$v^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}v_{\rho\sigma}, \quad v_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}v^{\rho\sigma}, \quad (19)$$

the vector (18) is constrained to be

$$v^{\mu\nu}v_{\mu\nu} = -v_{\rho\sigma}v^{\rho\sigma} = 0. \quad (20)$$

This restricts the matrix (17) to be singular one because in accordance with (18) the equation (20) reads as follows

$$v^{\mu\nu}v_{\mu\nu} = \varepsilon^{ij}\varepsilon_{kl}T_i{}^kT_j{}^l = 2 \det T = 0. \quad (21)$$

Thus we conclude that the rows and the columns of the matrix  $T$  are not independent, say

$$v_1^\mu v_{\mu i} = \alpha v_2^\mu v_{\mu i} \quad (22)$$

for  $\alpha$  being constant. On the other hand, for any spinor  $v_{\mu k}$  the identity

$$v^{\mu\nu}v_{\nu k} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{ij}v_{\rho i}v_{\sigma j}v_{\nu k} = 0 \quad (23)$$

can be derived since the  $SU(2)$  indices  $i, j, k$  run over two values only. Substituting the condition (22) in (23) we can rewrite equation (22) in the following form

$$(v_1^\mu - \alpha v_2^\mu)v_2^\nu v_{\nu k} = 0. \quad (24)$$

It is rather evident now that equation (22) has only one acceptable solution (17). The alternative one  $v_1^\mu = \alpha v_2^\mu$  is not reasonable since in virtue of this solution the vector  $v^{\mu\nu}$  turns out to be equal to zero identically.

Now we are ready to proceed to the construction of the twistor-like solutions of the equations (1)-(3) for the  $D = 6$  string. Here we shall use general conventions on the twistor basis in  $D = 6$  which turns out to be very useful, especially for getting the basis appropriated for the spinor harmonic variables.

We can say that the pair of spinors  $v_i^\mu, w_j^\mu$  forms the basis in six dimensions if they satisfy the following orthonormality conditions

$$v_{\mu i}w_j^\mu = v_i^\mu w_{\mu j} = \varepsilon_{ij}, \quad \varepsilon^{ij}(v_i^\mu w_{\nu j} + v_{\nu i}w_j^\mu) = -\delta_\nu^\mu. \quad (25)$$

Apparently, this notion is crucial for our further investigation. Indeed, let us consider the twistor-like solutions to the Virasoro constraints (2) in  $D = 6$

$$\partial_s x^{\mu\nu} = v^{\mu\nu}, \quad \partial_{\bar{s}} x^{\mu\nu} = u^{\mu\nu} = \varepsilon^{ij}u_i^\mu(\bar{s})u_j^\nu(\bar{s}), \quad (26)$$

with  $x^{\mu\nu}(s, \bar{s})$  and  $v^{\mu\nu}(s)$  defined in equations (14) and (18) respectively. It can be shown that in these notations the BNC conditions (3) for  $D = 6$  string may be represented in form

$$\varepsilon_{ij}\varepsilon_{kl}\varepsilon^{\mu\nu\rho\sigma}v_{\mu}^i v_{\nu}^j \partial_s v_{\rho}^k \partial_s v_{\sigma}^l = -16 q^2 \quad (27)$$

(the same is true for the spinor  $u_i^{\mu}(\bar{s})$ ). At the same time, taking account of (25) one can get the equation

$$\varepsilon_{ij}\varepsilon_{kl}\varepsilon^{\mu\nu\rho\sigma}v_{\mu}^i v_{\nu}^j w_{\rho}^k w_{\sigma}^l = 4. \quad (28)$$

We see that both of the equations (27) and (28) have the same structure. It suggests the following twistor-like solution to the BNC constraints in six dimensions

$$\partial_s v_i^{\mu} = 2 i q w_i^{\mu}(s), \quad \partial_{\bar{s}} u_i^{\mu} = 2 i \bar{q} \omega_i^{\mu}(\bar{s}). \quad (29)$$

Thus we obtain the set of equations

$$\begin{aligned} v_{\mu i} \partial_s v_j^{\mu} &= 2 i q \varepsilon_{ij}, & u_{\mu i} \partial_{\bar{s}} u_j^{\mu} &= 2 i \bar{q} \varepsilon_{ij}, \\ \varepsilon^{ij} (v_i^{\mu} \partial_s v_{\nu j} + v_{\nu i} \partial_s v_j^{\mu}) &= -2 i q \delta_{\nu}^{\mu}, \\ \varepsilon^{ij} (u_i^{\mu} \partial_{\bar{s}} u_{\nu j} + u_{\nu i} \partial_{\bar{s}} u_j^{\mu}) &= -2 i \bar{q} \delta_{\nu}^{\mu}, \end{aligned} \quad (30)$$

which provides the presence of the twistor-like basis in six dimensions similar to (4) and (13).

The prescriptions to be used in Section 2 can be straightforwardly generalized now to describe an embedding of the strings worldsheet into the  $D = 6$  flat space-time. Namely, the equation (7) acquires the form

$$\begin{aligned} u_i^{\mu} v_{\mu}^j &= G_i^j \equiv [\exp(-W)]_i^j, \\ G^{-1}{}_i^j (u_j^{\mu} v_{\nu}^i + u_{\nu j} v^{\mu i}) &= \delta_{\nu}^{\mu}. \end{aligned} \quad (31)$$

Note that the  $SU(2)$  matrix function  $W_i^j(s, \bar{s})$  that appears in (31) is not arbitrary. From the pseudoreality conditions (16) it follows that

$$\overline{G_i^j} = -\varepsilon^{ik} \varepsilon_{jl} G_k^l. \quad (32)$$

Equation (32) restricts the matrix  $W_i^j$  to take its values in the  $SU(2)$  group

$$W_i^j = W_0 \delta_i^j + i \vec{W} \vec{\sigma}_i^j \quad (33)$$

with  $W_0$  and  $\vec{W}$  being real. So, the new feature of the  $D = 6$  string contrarily to above mentioned examples is that the corresponding worldsheet function  $W(s, \bar{s})$  is represented by the quaternionic field (33).

To get the associated equation of motion, the following redefinition of the twistor variables will be done

$$v_i^{\mu+} = G^{-1/2} {}^j_i u_j^\mu, \quad v_i^{\mu-} = v^{\mu j} G^{-1/2}_{ji}. \quad (34)$$

Then equation (31) ensures the ordinary orthonormality properties of the harmonic variables (34), namely

$$\begin{aligned} v_i^{\mu+} v_\mu^{-j} &= \delta_i^j, \\ v_i^{\mu\pm} v_\mu^{\pm j} &= 0, \\ \varepsilon^{ij} (v_i^{\mu+} v_{\nu j}^- + v_{\nu i}^+ v_j^{\mu-}) &= \delta_\nu^\mu. \end{aligned} \quad (35)$$

The corresponding Cartan forms derived from (30), (34), (35) are

$$\begin{aligned} \Omega^{--} &\equiv v^{\mu-} dv_\mu^- = -2i q ds \tilde{G} G^{-1}, \\ \Omega^{++} &\equiv v^{\mu+} dv_\mu^+ = -2i \bar{q} d\bar{s} G^{-1} \tilde{G}, \\ \Omega^0 &\equiv v^{\mu+} dv_\mu^- = G^{-1/2} ds \partial_s G^{1/2} + G^{1/2} d\bar{s} \partial_{\bar{s}} G^{-1/2}, \end{aligned} \quad (36)$$

where  $\tilde{G} = \exp(-i\vec{W}\vec{\sigma})$  is the  $SU(2)$ -valued field. The 1-forms (36) can be viewed as a non-abelian generalization of the connection forms (10) from our earlier discussion. In accordance with their definition  $\Omega^{0,\pm\pm}$  satisfy the Gauss equation

$$d\Omega^0 - \Omega^{++}\Omega^{--} + \Omega^0\Omega^0 = 0 \quad (37)$$

which connects the fields in the r.h.s. of equation (36) with the following equation of motion

$$\partial_{\bar{s}}(\partial_s G^{-1/2} G^{1/2}) - \partial_s(\partial_{\bar{s}} G^{1/2} G^{-1/2}) + [G^{-1/2} \partial_s G^{1/2}, G^{1/2} \partial_{\bar{s}} G^{-1/2}] = 4q\bar{q} G^{-1} \tilde{G}^2 G^{-1}. \quad (38)$$

Thus, the dynamics of the  $D = 6$  string is described by the following pair of the equations of motion

$$\partial_s \partial_{\bar{s}} W_0 = 4q\bar{q} e^{2W_0}, \quad (39)$$

$$\partial_s(\partial_s \tilde{G}^{-1/2} \tilde{G}^{1/2}) - \partial_{\bar{s}}(\partial_{\bar{s}} \tilde{G}^{1/2} \tilde{G}^{-1/2}) + [\tilde{G}^{-1/2} \partial_s \tilde{G}^{1/2}, \tilde{G}^{1/2} \partial_{\bar{s}} \tilde{G}^{-1/2}] = 0. \quad (40)$$

Note that despite the difference in sign in the l.h.s. of the equations (37) and (12) the sign in r.h.s. of the equation (39) appears to be compatible with that of the Liouville equation (8). This is the result of the normalization condition (30) which in comparison with (4), (13) is supplemented by the multiplier  $i$ .



## 4 Conclusion

Thus we have demonstrated here that the constrained twistors principle enjoys a wide-spread support in the framework of the string theory. As opposed to the known geometrical approaches it provides us with the twistor-like variables compatible with the Virasoro and BNC constraints. These variables have the additional benefit allowing simultaneous treatment of the lower-dimensional strings after replacing the real  $SL(2, R)$  spinors by the  $SL(2, C)$  complex or  $SL(2, H)$  pseudoreal ones, one recovers the connection forms associated with the zero curvature representations of the three-, four- or six-dimensional strings. On the other hand, parametrizing these forms by the real, complex or quaternionic worldsheet function  $W(s, \bar{s})$  we obtain the new equation of motion for  $D = 6$  string besides the known Liouville equations. As far as we know, neither the Cartan forms (36) nor the equation (38) describing the dynamics of the  $D = 6$  string in such a minimal form have been obtained yet. Note that the  $SO(D - 2)$  nonlinear  $\sigma$ -model proposed in [5] could not be thought of as a minimal one because even in the case of  $D = 6$  it keeps a number of the redundant fields which could not be eliminated with the help of any reasonable dynamical principle.

In this article we have dwelled upon the bosonic strings only. All the important peculiarities of our approach are assumed to be applicable in the superstring theory. It can be proved, for instance, that to get a new version of the supersymmetric generalization of the Liouville equation given recently by Bandos, Sorokin and Volkov [5] it is sufficient to replace the constrained  $SL(2, R)$  twistors of Section 2 with its supersymmetric (anti)chiral superfields counterparts. We hope also that this direction of study may turn out to be instructive for a better understanding of the structure of the WZWN models related to the superstring theory [8, 9].

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